Letter

# Active jamming at criticality

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Jamming is ubiquitous in disordered systems, but the critical behavior of jammed solids subjected to active forces or thermal fluctuations remains elusive. In particular, while passive athermal jamming remains mean-field-like in two and three dimensions, diverse active matter systems exhibit anomalous scaling behavior in all physical dimensions. It is therefore natural to ask whether activity leads to anomalous scaling in jammed systems. Here, we use numerical and analytical methods to study systems of active, soft, frictionless spheres in two dimensions, and elucidate the universal scaling behavior that relates the excess coordination, active forces or temperature, and pressure close to the athermal jammed point. We show that active forces and thermal effects around the critical jammed state can again be captured by a mean-field picture, thus highlighting the distinct and crucial role of amorphous structure in active matter systems.

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*Introduction.* Disordered particulate systems are ubiquitous, from molecular glasses, colloidal suspensions, and foams to biological tissues and granular materials. Models of jammed solids have thus naturally attracted intense attention from both theorists and experimentalists alike [1–26]. It is by now well established that upon increasing its density, a system of athermal soft frictionless spheres generically develops a yield stress under which the solid responds elastically [4–12]. This jamming transition, known as "point $J$," is characterized by a critical packing fraction $\phi_c$, at which the jammed solids is marginally stable [1,4,6] and displays an isostatic contact network [6].

Intriguingly, the mechanical properties of jammed systems exhibit critical scaling close to point $J$ [27–29], and the corresponding upper critical dimension, $d_u$, is believed to be 2 [4,27,30–34]. Importantly, this has enabled researchers to successfully use mean-field theory to elucidate certain critical behavior of jamming for physically relevant dimensions ($d = 2, 3$).

Like jamming, active matter has received much attention over the past two decades [35–50]. Besides proving to be a fertile ground for novel physics, active matter offers quantitative descriptions of complex biological processes such as the dynamics of cellular tissues in health and diseases or embryogenesis [26,51–61]. In contrast to athermal jamming, mean-field theory seems to *always* break down in *all* physical dimensions in diverse active systems at criticality and in the symmetry-broken phases [62–69]. Therefore, it is natural to ask whether adding activity to static jammed packings would alter the mean-field picture that governs its critical behavior.

While jamming under activity has recently been explored, existing works focus on the glassy regime away from the critical point [70–79], such as aging dynamics in active glasses [80–84] and connections between dense active systems and sheared athermal passive jammed solids [85–89], mirroring the earlier effort to recast shear induced fluctuation into an effective temperature in passive jammed systems [13,90–92]. This is surprising since jamming has long been viewed as an end point of the glassy phase [2,4,93]. While recent studies have clarified the important differences between jamming and glass transitions for passive systems [11,34,93–96], this has however not been done for active systems.

Interestingly, systems of active Brownian particles (ABP) at high density have been shown to jam intermittently and that the lifetimes of these transiently jammed states lengthens with the persistence of the particles [97]. Since the lifetimes of these transient jammed states can be arbitrarily long, they are clearly experimentally relevant. However, the associated scaling behavior remains unexplored. In this Letter, we fill this void by elucidating the scaling behavior of these transiently jammed states under both activity and thermal fluctuations around the static critical jamming point.

*Scaling ansatz.* As passive and athermal jamming is a critical phenomenon for which the critical point is exactly at isostaticity (i.e., the average coordination $z$ is $z_{\rm iso} = 2d$), analysis based on scaling ansatz provides scaling relations among diverse physical observables [27]. Here, we also start by formulating a scaling ansatz incorporating the effects of active forces in the athermal jamming scenario. Specifically, we consider a generic system of soft frictionless spherical particles above jamming onset, i.e., with pressure $p > 0$. Here, we assume that the particles interact via a purely repulsive

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spring potential

$$U(r_{ij}) = \frac{\varepsilon}{\alpha}\left(1 - \frac{r_{ij}}{\sigma_{ij}}\right)^{\alpha}\Theta(\sigma_{ij} - r_{ij}), \tag{1}$$

where $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ is the distance between particles $i$ and $j$, $\sigma_{ij} = (\sigma_i + \sigma_j)/2$ and $\Theta(x)$ is the Heaviside function. In the absence of active forces, the following scaling relation between $\Delta z \equiv z - z_{\mathrm{iso}}$ and $p$ is well established [6,27]:

$$\Delta z \sim p^{1/[2(\alpha-1)]}, \tag{2}$$

which, following small corrections to scaling as expected in the upper critical dimension, leads to $\Delta z \sim p^{1/1.88}$ for 2D systems with harmonic interactions ($\alpha = 2$) [27].

We now imagine that the particles can exert a persistent active force $f$ in a randomly chosen direction. Since prior work has demonstrated that active forces have the generic effects of unjamming a system, we anticipate that $\Delta z$ should decrease with $f$. We also expect the system to revert back to the same inherent structure when active forces are removed as long as $\Delta z$ remains positive. In other words, we expect the system to only explore locally its potential energy landscape and to remain in the basin of attraction of its initial static configuration which is justified by the results of Ref. [97] and our own simulations.

Around this critical region where $\Delta z$, $p$, and $f$ are all small, we expect the system to be scale-invariant with a single lengthscale controlled by one of the control parameter [98]; we choose our control parameter to be $p$ since it is the physical quantity that we actually control in most simulation studies. As such, we write the following scaling ansatz:

$$\Delta z = p^{1/[2(\alpha-1)]} S(f/p^{\chi}), \tag{3}$$

where $S$ is a *universal* scaling function such that $\lim_{x\to 0} S(x)$ is a positive constant to recover the static scaling relation between $\Delta z$ and $p$ (2).

Importantly, we anticipate that the scaling function $S(x)$ is monotonically decreasing w.r.t. $x$, and eventually becomes 0 at $x_c$. In other words, at a critical force $f_c$ such that

$$f_c = x_c p^{\chi}, \tag{4}$$

$\Delta z$ is exactly zero in the system. We now test this scaling ansatz via means of numerical simulations.

*Model and simulations.* We study the dynamics of two-dimensional jammed packings of $N$ spheres interacting via purely repulsive forces when subjected to active forces and thermal fluctuations. Our numerical simulations focus on the case of harmonic interactions [$\alpha = 2$ in Eq. (1)]. We create static jammed packings at fixed pressures $p$ to control the packings distance to the athermal jamming transition point. To do so, we start with random configurations of discs at high volume fraction ($\phi = 0.95$) in a square box of size $L$ with periodic boundary conditions. We then follow the fast inertial relaxation engine (FIRE) energy minimization procedure until the maximum unbalanced force on any particle is less than $f_t = 10^{-14}$. Following each energy minimization, we either increase (if the pressure is below target) or decrease (if the pressure if above target) the diameter of the particles proceeding to a bisection until we obtain a mechanically stable configuration within 1% of the target pressure $p$. To avoid crystallization, we work with 50 : 50 binary mixtures of particles with diameter ratio 1 : 1.4 as is typically done in studies of disordered solids [6,8].

These mechanically stable configurations form the initial conditions of our dynamical simulations. In what follows, we subject our jammed packings to three different perturbations:

(i) *Persistent active forces using an ABP model* in which the positions of the particles are governed by an overdamped Langevin equation of the form

$$\dot{\mathbf{r}}_i = \frac{1}{\zeta}\left[-\sum_{j=1}^{N}\nabla_i U(r_{ij}) + f\hat{\mathbf{e}}_i\right], \tag{5a}$$

$$\dot{\theta}_i = \sqrt{2D_r}\xi_i, \tag{5b}$$

where $\hat{\mathbf{e}}_i = (\sin\theta_i, \cos\theta_i)$ is the direction of self-propulsion, $f$ the strength of active force and $\zeta$ a friction coefficient. The dynamics of the self-propulsion direction is governed by rotational diffusion with coefficient $D_r$ and $\xi_i$ is a noise term defined below.

(ii) *Thermal fluctuations* via Brownian dynamics in which the positions of the particles are governed by an overdamped Langevin equation of the form

$$\dot{\mathbf{r}}_i = \frac{1}{\zeta}\left[-\sum_{j=1}^{N}\nabla_i U(r_{ij})\right] + \sqrt{2D}\boldsymbol{\eta}_i, \tag{6}$$

where the translational diffusion coefficient $D$ and the friction coefficient $\zeta$ set the temperature $T$ via the Stokes-Einstein relation and $\boldsymbol{\eta}_i$ is a noise term defined below.

(iii) *Persistent active forces using an AOUP model* which bridge the gap between our active Brownian particles simulations and our passive Brownian dynamics. Here, the positions of the particles are governed by

$$\dot{\mathbf{r}}_i = \frac{1}{\zeta}\left[-\sum_{j=1}^{N}\nabla_i U(r_{ij}) + \mathbf{f}_i\right], \tag{7}$$

where $\mathbf{f}_i$ denotes the active forces. While ABPs are subject to active forces with constant amplitudes and diffusing directions, AOUPs are subject to varying amplitude active forces which are governed by

$$\tau_a\dot{\mathbf{f}}_i = -\mathbf{f}_i + \sqrt{2D_a}\boldsymbol{\psi}_i. \tag{8}$$

Here, $\tau_a$ is the persistence time of the active forces, $D_a$ a diffusion constant, and $\psi_i$ is a noise term defined below. We fix the diffusion coefficient $D_a$ and vary $\tau_a$ to change the strength of active force, defined in the AOUP model as $f = \sqrt{2D_a/\tau_a}$.

Finally, $\boldsymbol{\eta}_i$, $\xi_i$, and $\boldsymbol{\psi}_i$ are all zero-mean, unit variance Gaussian random variables such that

$$\langle \eta_{i,\alpha}(t)\rangle = 0, \quad \langle \eta_{i,\alpha}(t)\eta_{j,\beta}(t')\rangle = \delta_{i,j}\delta_{\alpha,\beta}\delta(t-t'), \tag{9a}$$

$$\langle \xi_i(t)\rangle = 0, \quad \langle \xi_i(t)\xi_j(t')\rangle = \delta_{i,j}\delta(t-t'), \tag{9b}$$

$$\langle \psi_{i,\alpha}(t)\rangle = 0, \quad \langle \psi_{i,\alpha}(t)\psi_{j,\beta}(t')\rangle = \delta_{i,j}\delta_{\alpha,\beta}\delta(t-t'), \tag{9c}$$

with $(i, j) \in [1, N]$ and $(\alpha, \beta) \in \{x, y\}$.

We nondimensionalize the equations of motion using the smaller particle diameter $\sigma$ and potential energy $\varepsilon$ as basic units of length and energy. In ABP simulations, the typical timescale is given by the persistence time $\tau_p = D_r^{-1}$, similarly

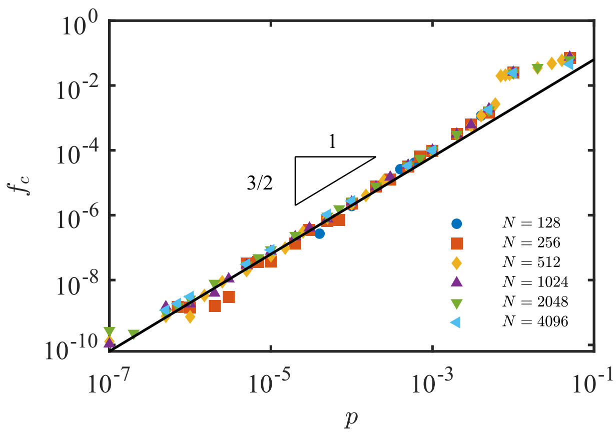


FIG. 1. Scaling of the critical active force ($f_c$) vs pressure ($p$). We define $f_c$ as the active force at which an initially overjammed system (with pressure $p$) becomes isostatic. The scaling behavior found for ABPs for various system sizes $N \in [128, 4096]$ is consistent with a power-law scaling with exponent $\chi = 3/2$ (solid black line).

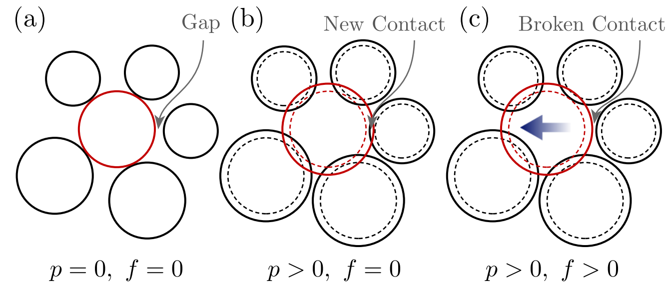


FIG. 2. Schematics illustrating our heuristic analytical argument. (a) Particle configuration at critical jamming under no active force. The gap distribution between nontouching neighbors follows the scaling law in Eq. (11). (b) Upon further compression (or particles' expansion), the pressure $p$ becomes positive and new contacts are formed as gaps close. The new particles' profiles are shown in solid lines whereas the pre-compressed profiles are in dashed lines. (c) When active forces are switched on ($f > 0$), contacts can be broken again as the active force (blue arrow) of the red particle can counteract the steric interactions with its neighbors.

in AOUP simulations where the persistence time is $\tau_a$, while in Brownian dynamics simulations, a more natural choice for the timescale is given by $\tau = \sigma^2/D$. In all simulations, we set $\sigma = 1$, $\varepsilon = 1$, and $\zeta = 1$. Further, the rotational diffusion coefficient $D_r$ is taken to be $10^{-2}$ unless stated otherwise. To discard any potential finite size effect in our results, we varied the number of particles in the system from $N = 128$ to $N = 4096$. The target pressures of the initial configurations were taken in the range $[10^{-7}, 10^{-1}]$. Finally, results are averages over 100 independent realizations.

To estimate $\chi$ using Eq. (4), we start with static packings above jamming onset with different pressure values $p$ in the absence of active forces; then increasing the active force strength, we find the critical value of the active force for which the time-averaged excess coordination $\Delta z$ becomes zero. Figure 1 shows that

$$\chi = 1.5 \pm 0.1. \tag{10}$$

We now present a heuristic argument to justify the value of $\chi$ obtained numerically.

*Heuristic argument for estimating* $\chi$. As unjamming is generically controlled by the excess coordination $\Delta z$ under active forcing, we expect the critical active force to be dictated by the statistics of interparticle contacts in the overjammed static configurations. The scaling behavior of the distribution of interparticle gaps $h$ has been studied both analytically and numerically [31,34,99–102]. Specifically, we have in the vicinity of the athermal jamming point

$$P(h) \sim h^{-\gamma}. \tag{11}$$

In our soft sphere simulations, we obtain $\gamma = 0.42 \pm 0.04$ [103] which is close to the value of $\gamma_\infty = 0.41269...$ predicted in the infinite-dimensional mean-field theory of the hard sphere glass transition [101,104,105]. In our model of soft spheres, the gap distribution shows a weak dependence on the pressure, which we neglect here as it does not affect our argument (within the precision of our numerical estimate).

We now assume that the gap distribution remains the same at small enough pressures $p$ and active forces $f$. As $p$ increases in the absence of active forces, gaps close and new contacts form (Fig. 2). This increase in new contacts is given by

$$\Delta z \sim \int_0^\delta h^{-\gamma} dh \sim \delta^{1-\gamma}, \tag{12}$$

where $\delta$ is the average overlap. For our harmonic soft spheres simulations, $\delta$ is known to scale like $p$ in the overjammed regime [101], we thus conclude that $\Delta z \sim p^{0.58}$, a scaling exponent already in good agreement with Eq. (2). For more general interaction potentials (1), we expect the pressure to scale as $p \sim \delta^{\alpha-1}$.

If each particle is subject to an active force $f$, the average force needed to eliminate all newly formed contacts and thus bring the system back to isostaticity is

$$f_c \sim \int_0^\delta hP(h)dh \sim \delta^{2-\gamma} \sim p^{(2-\gamma)/(\alpha-1)}, \tag{13}$$

where the upper-bound $\delta$ again corresponds to the elimination of $N\Delta z$ contacts. For our harmonic soft spheres, we obtain that $f_c \sim p^{1.58}$. This heuristic argument thus supports the value of $\chi$ (10) obtained from simulation.

*Universal scaling function S.* This new scaling allows us to elucidate the form of the universal scaling function $S$ numerically. As presented in Fig. 3(a) and Ref. [103], $S(x)$ is monotonically decreasing with $f$ for all pressures. Provided the scaling behaviors given in Eqs. (2) and (4), we attempt to collapse our data to obtain $S(x)$. In particular, we can scale the curves in the low activity regime as the excess coordination scales as $p \sim \Delta z^{1.88}$. Further, using the scaling of the critical active force with pressure, we obtain a good collapse of $\Delta z$ as a function of $f$ for all pressures [Fig. 3(a)]. This data collapse supports the existence of scaling relations and of universal behavior in the vicinity of the jamming point (low enough pressures).

Before going further, we need to discuss the conditions under which measuring the critical active force $f_c$ is meaningful. In using Eq. (4) to obtain the scaling exponent $\chi$, we

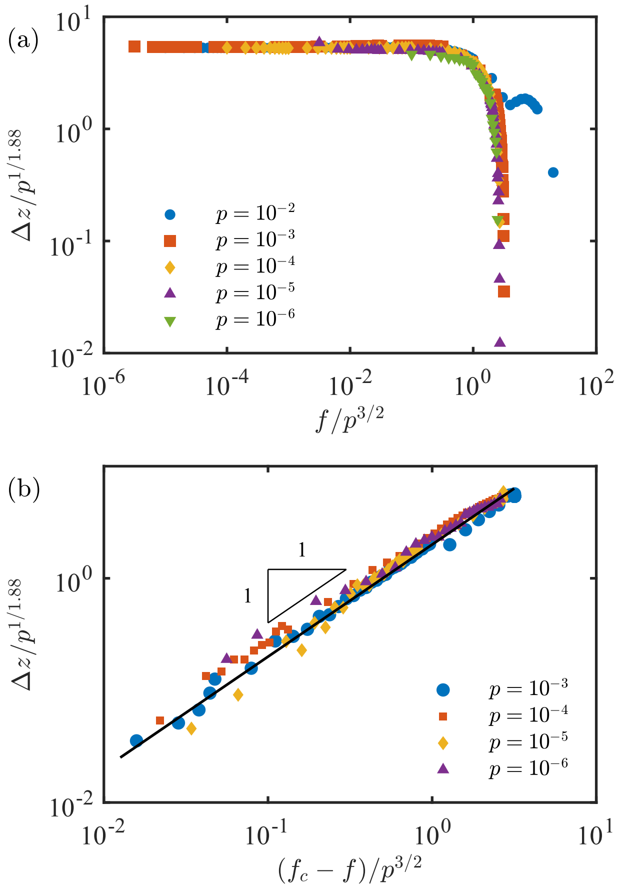


FIG. 3. Universal scaling function $S(x)$. (a) Collapsed data for excess number of contacts per particle ($\Delta z$) as a function of the active force $f$ at various pressure values for $N = 2048$. A good collapse is obtained via a rescaling of $\Delta z$ by $p^{\beta}$ (with $\beta = 1.88$ for $d = 2$, see Ref. [27]) and $f$ by $p^{\chi}$ with $\chi = 3/2$. (b) Evolution of the excess coordination number $\Delta z$ near the critical active force for various values of pressure for $N = 2048$. The black solid line shows a linear scaling.

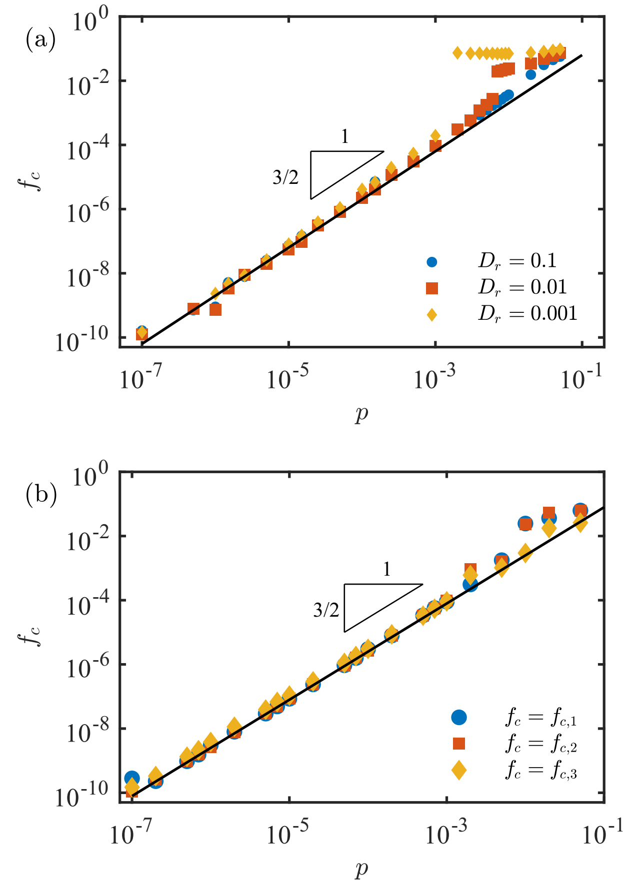


FIG. 4. Effect of persistence and thermal fluctuations. (a) Critical active force as a function of pressure for various values of the rotational diffusion constant $D_r$ in simulations of $N = 512$ active Brownian particles (ABP). The solid black line shows a power law scaling with exponent $\chi = 3/2$. For lower values of $D_r$, the self-propulsive forces become more persistent and the regime of validity of our power-law scaling is limited to lower values of the pressure. (b) Comparison of critical forcing for three perturbation protocols for systems of size $N = 2048$: (i) persistent active forces with constant magnitude but diffusive direction in active Brownian particles simulations (ABP), (ii) thermal fluctuations in Brownian dynamics simulations (BD), and (iii) persistent active forces with fluctuating magnitude and direction in active Ornstein-Uhlenbeck particles simulations (AOUP). The critical forcing follows in all three cases the same scaling behavior as a function of $p$ close to the passive athermal jamming point ($p \to 0$). The solid black line shows a power law scaling with exponent $\chi = 3/2$.

focused on the vanishing point of $S$ (i.e., the point where $\Delta z = 0$). However, as $\Delta z$ becomes negative (i.e., below isostaticity), the system loses rigidity and so our scaling ansatz cannot be correct in this regime. In other words, it is unclear whether $S$ remains continuous as it vanishes. Fortunately, left continuity of $S$—which corresponds to $dS(x)/dx|_{x=x_c^-}$ being well-defined—is all that is required for the above argument to work.

In general, this condition implies the following scaling relation for $f$ close to $f_c$:

$$\frac{\Delta z}{p^{1/[2(\alpha-1)]}} \sim A \times \left(\frac{f_c - f}{p^{\chi}}\right), \tag{14}$$

where $A \equiv -dS(x)/dx|_{x=x_{c,-}}$ is a positive constant. In the case of our 2D systems of harmonic soft spheres, we expect the following scaling relation $\Delta z/p^{1/1.88} \sim A \times (f_c - f)/p^{\chi}$, again the value of 1.88 is due to small corrections in the upper critical dimension [27]. Figure 3(b) shows that this scaling relation is indeed satisfied; this implies the left-continuity of $S$ at the vanishing point, justifying our procedure to determine $\chi$ (Fig. 1).

*Scaling behavior and persistence.* Studies of athermal jammed packings define jammed configuration as having a positive excess coordination. With active forcing, we chose to extend this criterion and define jammed systems as systems whose *steady-state time-averaged* excess coordination is positive. As our packings coordination may fall below isostaticity transiently, they may transiently unjam before rejamming. To analyze this, we studied (i) distributions of Debye-Waller factors, (ii) mean-square displacements, and (iii) intermediate scattering functions in packings under active forces. Strikingly, all quantities display signatures of the critical transition at $f = f_c$ and show that structural rearrangements are rare even on timescales which are long compared to the active force persistence time $\tau_p = D_r^{-1}$ [79,103].

After focusing on the long persistence time regime, we study the robustness of the scaling regime as the persistence of the active forcing decreases. Figure 4(a) shows that the scaling behavior persists even as the persistence time goes down by two orders of magnitude. Interestingly, we observe that as $\tau_p$ decreases, the regime of validity of the critical force scaling shifts towards larger pressure values (as evidenced by the shoulders developing in the high $p$ limit), while scaling (4) remains intact in the vicinity of the jamming point.

*Active forcing versus thermal fluctuations.* As we have seen that the scaling regime remains robust even for very small persistence lengths, we extend our analysis to the situation where jammed packings are subjected to thermal fluctuations (zero persistence time limit) and ask the question of whether the *same* scaling can be measured robustly in this case. Although a jammed system will inevitably melt under thermal fluctuations in the long-time limit, we will test the robustness of the scaling regime in the intermediate time regime (under the same simulation and measurement protocols used so far).

To study thermal fluctuations, we follow dynamically the network of contacts in Brownian dynamics simulations. We identify the critical temperature $T_c$ at which the time-averaged excess coordination $\Delta z$ vanishes. To compare directly the effects of thermal fluctuations and active forces, we define our critical forcing as $f_{c,2} = \sqrt{2T_c}$. Strikingly, Fig. 4(b) shows that the critical thermal forcing $f_{c,2}$ follows the *same power-law scaling with pressure* as its active counterpart (here, in systems with $N = 2048$ particles; see Ref. [103] for a system size analysis). Finally, we also verify that we obtain the same scaling for AOUPs for which we define the critical force as $f_{c,3} = \sqrt{2D_a/\tau_a}$, where $\tau_a$ is the persistence time of the active force magnitude. We thus conclude to the existence of dynamically jammed states even at finite forcing whose scaling behavior is independent of the driving mechanism in the vicinity of point $J$.

*Summary and outlook.* In summary, we have studied the effects of active forces and thermal fluctuations on long-lived jammed states of soft frictionless particles in two dimensions close to the critical athermal jamming point. Using numerical simulations supplemented by a heuristic analytical argument, we elucidated the universal scaling relations between the excess coordination, the active force (or temperature), and pressure. In particular, the heuristic, mean-field based argument that we used to support the scaling exponent suggests that the mean-field picture applicable to athermal jamming continues to hold. This is in surprising contrast to diverse active matter systems in which anomalous scaling behavior is the norm. An interesting future direction would be to seek the existence of diverging length- or timescales.

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